

Given the following matrices compute the indicated quantities:

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 9 & -6 \\ 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 0 & 3 \\ 0 & 7 & -1 \\ 0 & 0 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 9 & 5 & 4 \end{bmatrix}$$
$$D = \begin{bmatrix} -2 & 0 & -4 & 1 \\ 1 & 0 & -1 & 6 \\ 8 & 2 & 1 & -1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) $(AB)^{-1}$ using analytic solution

(b) C⁻¹, E⁻¹ using LU decomposition

(c) $(DD^T)^{-1}$, $(D^TD)^{-1}$ using Gaussian elimination



Use the inverse of the coefficient matrix to solve the following systems

$$3x_1 + x_2 = 6 \qquad -2x_1 + 4x_2 - 3x_3 = -1 -x_1 + 2x_2 + 2x_3 = -7 \qquad 3x_1 - 2x_2 + x_3 = 17 5x_1 - x_3 = 10 \qquad -4x_2 + 3x_3 = -9$$



Solve the following systems using Gauss Jordan – Gauss elimination - LU decomposition methods.

$$\begin{array}{ll} x_1 - 3x_2 + 4x_3 = 12 & x_1 - 3x_2 + 4x_3 = 0 \\ 2x_1 - x_2 - 2x_3 = -1 & 2x_1 - x_2 - 2x_3 = 5 \\ 5x_1 - 2x_2 - 3x_3 = 3 & 5x_1 - 2x_2 - 3x_3 = -8 \end{array}$$



Solve the linear system by Gauss-Seidel Method, Cholesky decomposition

- 1) x + 4y + z = 2, 4x + y + z = 5, x + y + 4z = 3
- 2) x + y + 5z = 3, 6x 2y + 2z = 5, 3x + 9y + 4z = 11



1- Use Picard's method up to third approximation the following differential equation $y^{*} = x^3 (y^* + y), y(0) = 1, y^{*}(0) = \frac{1}{2}.$

2- Find y(0.3) for the D.E. $y^{*} = 3x + y^{2}$, y (0) = 1 using Euler method, h=0.1.

3- Solve the following differential equation $y^{*} = x^{2}$, y^{2} , y(0) = 0, using Taylor method

Assignment 6

Solve the following differential equations using Runge Kutta of 2^{nd} , 4^{th} order – Euler– Taylor – Picard methods to find y(0.2).

i)

$$\frac{d^2 y(t)}{dt^2} + 100 \frac{dy(t)}{dt} + 10^4 y(t) = 10^4 |\sin(377t)|$$

y(0) = 7, y(0) = 3

ii)

$$\frac{dx}{dt} = -10(x-y)$$
$$\frac{dy}{dt} = -xz + 28x - y$$
$$\frac{dz}{dt} = xy - 8z/3$$

x(0) = 2, y(0) = -1, z(0) = 3, h = 0.1



- 1- Find Lagrange interpolating polynomial satisfy (1,3), (5,-7), (-13,4), (2,47)
- 2- Use Newton's method to find one of the three roots of the cubic polynomial $4x^3 15x^2 + 17x 6 = 0$.



- 1- Find Parabolic equation that fit (3,5), (15,114), (19,201), (23,330)
- 2- Find the constants of the curve $y = a\cos x + b \ln x + c e^{x}$ that fit (1,3), (5,14),

(19,101)



I) Solve the following heat equations using finite difference method:

1)
$$U_{xx} = U_t$$
, $U(0,t) = U(1,t) = 0$, $U(x,0) = \begin{cases} 2x & 0 \le x \le 1/2 \\ 2(1-x) & 1/2 \le x \le 1 \end{cases}$ $0 \le x \le 1, 0 \le t$

2) $U_t + U_{xx} = 0$, U(0,t) = U(1,t) = 0, $U(x,0) = \sin \pi x$ $0 \le x \le 1, 0 \le t$

Take h = 0.1, k = 0.01 for the above problems.

II) The temperature u(x,t) of a long, thin rod of constant cross section and homogeneous conducting material is governed by the one dimensional heat equation. If heat is generated in the material by resistance to current or nuclear reaction, the heat equation becomes: $U_{xx} + \frac{kr}{\rho c} = k U_t$, $0 < x < \ell$, 0 < t, where ℓ is the length, ρ is the density, c is specific heat, k is the thermal diffusivity of the rod. The function r = r(x,t,u) represent the heat generated per unit volume. Suppose $\ell = 1.5 \text{ cm}$, k = 1.04 cal/cm.deg.s, $\rho = 10.6g/\text{ cm}^3$, c = 0.056 cal/g.deg. and $r = 5 \text{ cal/ cm}^3$. If the ends of the rod are kept at 0° c, then U(0,t) = U(ℓ ,t) = 0, t > 0 and the initial temperature distribution is given by U(x,0) = $\sin(\frac{\pi x}{\ell})$, $0 \le x \le \ell$.

Take h = 0.15, k = 0.0225



I) Solve the following wave equations using finite difference method:

1) $U_{tt} - 4U_{xx} = 0$, U(0,t) = U(1,t) = 0, $U(x,0) = \sin \pi x \& U_t(x,0) = 0$, $0 \le x \le 1$, $0 \le t$

2)
$$U_{tt} - \frac{1}{16\pi^2}U_{xx} = 0$$
, $U(0,t) = U(0.5,t) = 0$, $U(x,0) = 0$ & $U_t(x,0) = \sin(4\pi x)$,

$$0 \le x \le 0.5, 0 \le t.$$

Take h = 0.1, k = 0.01 for the above problems.

II) In an electric transmission line of length ℓ that carries alternating current of high frequency (called a "loss less" line). The voltage v and current i are described by:

$$\frac{\partial^2 v}{\partial x^2} = Lc \frac{\partial^2 v}{\partial t^2}, \qquad \frac{\partial^2 i}{\partial x^2} = Lc \frac{\partial^2 i}{\partial t^2}, \ 0 \le x \le \ell, 0 \le t$$

L is the inductance per unit length = 0.3 henries / ft and C is the capacitance per unit length = 0.1 farad/ft. suppose voltage v and current i also satisfy:

$$v(0,t) = v(200,t) = 0$$
, $0 < t$, $v(x,0) = 110sin(\frac{\pi x}{200})$, $\frac{\partial v}{\partial t}(x,0) = 0$, $0 \le x \le 200$,

similarly

$$i(0,t) = i(200,t) = 0$$
, $0 < t$, $i(x,0) = 5.5 \cos(\frac{\pi x}{200})$, $\frac{\partial i}{\partial t}(x,0) = 0$, $0 \le x \le 200$.

Take h = 10, k = 0.1



I) Solve the following elliptic equations using finite difference method:

1) $U_{xx} + U_{yy} = 4$, with B.C. $U(0,y) = y^2$, $U(1,y) = (y-1)^2$, $0 \le y \le 2$ & $U(x,0) = x^2$, $U(x,2) = (x-2)^2$, $0 \le x \le 1$. 2) $U_{xx} + U_{yy} = xe^y$, with B.C. U(0,y) = 0, $U(2,y) = 2e^y$, $0 \le y \le 1$ & U(x,0) = x, $U(x,1) = e^x$, $0 \le x \le 2$. Take h = 0.2, k = 0.2 for the above problems.

II) A 6 cm by 5 cm rectangular silver plate has heat being uniformly generated at each point at the rate q = 1.5 cal / cm³.s. Let x represent the distance along the edge of the plate of length 6 cm and y be the distance along the edge of the plate of length 5 cm. Suppose the temperature u along the edges is kept at the following temperatures: u(x,0) = x(6-x), u(x,5) = 0, $0 \le x \le 6$ and u(0,y) = y(5-y), u(6,y) = 0, $0 \le y \le 5$, where the origin lies at a corner of the plate with coordinates (0,0) and edges lies along positive x and y axis. The steady state temperature u(x,y) satisfy Poisson equation: $U_{xx} + U_{yy} = q / k$, where k is the thermal conductivity is 1.04 cal/cm deg.s, h = 0.4, k = 1/3.

Assignment 12



Determine

- i) Degree of vertices
- ii) Trail-path-walk
- iii) Incidence matrix
- iv) Adjacency matrix
- v) Eulerian trail or Eulerian circuit



Determine which of the following Eulerian circuit or Eulerian path or Hamiltonian circuit or Hamiltonian path.



Student Name: Date: Grade: 18



For the network below, where the numbers by each arc represent the cost of removing the arc, determine Incidence matrix & Adjacency matrix

